GENERATION OF HIGHER ELECTRON CYCLOTRON FREQUENCY HARMONICS IN PLASMA WITH A BEAM

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We examine nonlinear excitation of the higher electron-cyclotron frequency harmonics for waves propagating perpendicular to an external uniform magnetic field in a Maxwell plasma for the case of low-density electron beam passage through the plasma. It is shown that the nonlinear excitation mechanism leads to the possibility of generating cyclotron harmonics for plasma parameters for which generation does not occur from the linear theory viewpoint. The nonlinear cyclotron harmonic generation increments are calculated for nonlinear scattering by the beam and plasma electrons of the high frequency longitudinal waves excited in the plasma by the beam.

Study of electron gyrofrequency harmonic propagation is of interest, first of all, for cyclotron heating of plasma, the radiation of cyclotron harmonics by nonequilibrium plasma, and for their interaction with other types of waves. Because of the absence of Landau damping for such waves when they propagate perpendicular to the external magnetic field, the energy contained in such waves may be very large, and all possible nonlinear interactions with participation of these waves become quite important. Therefore, we should examine the possibilities of both linear and nonlinear excitation of such waves. Nonlinear generation of electron cyclotron harmonics in a plasma with current was examined in [1]; also presented there are references to experimental studies. On the other hand, generation of electron gyrofrequency harmonics has been observed in a plasma when passing a low-density electron beam through it (for example, [2]). In the present paper we examine a possible nonlinear mechanism for such generation. The linear mechanism for excitation of such waves developed in several papers (for example, [3-5]) leads to limitations on the beam and plasma parameters. The excitation of quasilongitudinal electronic cyclotron waves is examined in [3, 4] under the assumption that the electron velocity distribution function has the form

$$f(v_\perp, v_\parallel) = \frac{n_1}{2\pi v_{0\perp}} \delta(v_\perp - v_{0\perp}) \delta(v_\parallel - v_{0\parallel})$$

and in [5], along with the distribution in the form of the delta function, there are background electrons with Maxwellian distribution function.

Essential for the possibility of generation is that the transverse velocity distribution function be non-Maxwellian; increase of the transvere velocity spread leads to stabilization [4]. In all cases there is a lower generation threshold $\omega_L/\Omega > 1$ ($\omega_L = (4\pi e^2 N/m_e)^{1/2}$ is the electron Langmuir frequency, $\Omega = |e|H/m_e c$ is the electron cyclotron frequency), depending on the cyclotron harmonic number, and also in the presence of Maxwellian electrons (we shall term them the plasma proper in contrast with the beam with delta function distribution) on the ratio of the densities and characteristic velocities of the beam and plasma [5]. For each value of $q = \omega_L/\Omega$ above the threshold there are ranges of values of $\lambda = k_1 v_{0\perp}/\Omega$ (where k_1 is the wave number of the electron cyclotron wave) in which there is no generation (specifically, in all cases there is no generation for $\lambda < 1$). The nonlinear excitation mechanism examined below leads to the possibility of generation of both quasilongitudinal and ordinary and extraordinary electron cyclotron waves during scattering by the plasma and beam electrons of the quasilongitudinal

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high-frequency waves excited in the plasma in the magnetic field by the low-density electron beam $n_1 \ll n_0$ (n_1 is the beam density, n_0 is the plasma density), whose velocity exceeds the phase velocity v_F of these waves [6, 7]. This generation occurs even for Maxwellian distribution of the transverse beam velocities in the case of corresponding longitudinal beam velocities. Generation can occur in both dense $q \gg 1$ and non-dense $q \ll 1$ plasma. The ranges of values of the parameter $\mu_1 = (k_1 v_e/\Omega)^2$ are determined by the closeness of the generated frequency ω_L to the harmonics of the electron gyrofrequency $\nu_0 \Omega$.

1. BASIC EQUATIONS

Let the plasma and beam electrons be characterized by Maxwellian distributions

$$f_0(\mathbf{v}) = \frac{n_0}{(2\pi)^{3/2} v_e^3} \exp\frac{-\mathbf{v}^2}{2v_e^2}, \qquad f_{01}(\mathbf{v}) = \frac{n_1}{(2\pi)^{3/2} u_e^3} \exp\frac{-(\mathbf{v}-\mathbf{v}_0)^2}{2u_e^2} \tag{1.1}$$

Here n_0, v_e are the density and average thermal velocity of the plasma electrons, n_1, u_e are the same quantities for the beam, v_0 is the average systematic velocity of the beam, and H_0 is the external constant and uniform magnetic field. We shall consider that $n_1 \ll n_0$, and $v_0 \parallel H_0$.

For sufficiently high beam velocity $v_0 > v_F$ high frequency longitudinal oscillations are excited in the plasma with the frequencies [6]

$$\omega_\pm^2(\theta) = {}^1/_2(\omega_L^2 + \Omega^2) \pm {}^1/_2 \sqrt{(\omega_L^2 + \Omega^2)^2 - 4\omega_L^2\Omega^2\cos^2\theta} \tag{1.2}$$

Here θ is the angle between the wave vector **k** and the magnetic field H_0. In deriving (1.2) the plasma was assumed cold, i.e., the conditions were satisfied

$$\mu = \frac{k_\perp^2 v_e^2}{\Omega^2} \gg 1, \qquad \beta_n = \frac{\omega - n\Omega}{|k_z| v_e} \gg 1 \tag{1.3}$$

In the limiting cases of plasma of very high and very low density ($q \gg 1$, $q \ll 1$), we have from (1.2)

$$\omega_+ \approx \omega_L(1 + {}^1/_2 q^2 \sin^2\theta), \ \omega_- \approx \Omega \operatorname{sos}\theta \ (q \gg 1) \tag{1.4}$$

$$\omega_+ \approx \Omega\ (1 + q^2/2 \sin^2\theta), \ \ \omega_- \approx \omega_L \cos\theta \ \ (q \ll 1) \tag{1.5}$$

The formulas for ω_+ are valid to within terms of order m_e/m_i for any angles θ; the formulas for ω_- are valid provided

$$|1/2\,\pi - \theta| \gg (m_e / m_i)^{1/2} \tag{1.6}$$

Moreover, since ω_+ or $\omega_- \to \Omega$ as $\theta \to 0$, the following condition must be satisfied:

$$\frac{kv_e}{\Omega} \ll \frac{q^2}{2|1-q^2|}\theta^2$$

The nonlinear equation describing the process of induced scattering by the plasma and beam electrons can be obtained from semiquantum balance arguments [8]

$$\frac{\partial N_{\mathbf{k}_1}^{\sigma'}}{\partial t} = -\sum_{\nu\alpha}\int w_\nu^{\sigma\sigma'\alpha}(\mathbf{p}_\alpha, \mathbf{k}, \mathbf{k}_1) N_{\mathbf{k}}^{\sigma} N_{\mathbf{k}_1}^{\sigma'} \left(k_{2z}\frac{\partial f_{\mathbf{p}}}{\partial p_z} + \frac{\nu\Omega}{v_\perp}\frac{\partial f_{\mathbf{p}}}{\partial p_\perp}\right) d\mathbf{p}\, d\mathbf{k} \tag{1.7}$$

Here $N_{\mathbf{k}}^\sigma$ is the number of quanta of sort σ, $w_\nu^{\sigma\sigma'\alpha}$ is the probability of scattering of the wave σ with momentum **k** by the particle α with momentum $\mathbf{p}_\alpha$ with transformation into the wave σ' with momentum $\mathbf{k}_1$. Both the plasma electrons and the beam electrons contribute to the scattering probability. However, the beam contribution is on the order of $n_1/n_0 \ll 1$ of the plasma electron contribution, and therefore we shall neglect the beam contribution hereafter. However, f_p is the overall distribution function of the beam and plasma electrons. The expression for the scattering probability has the form

$$\begin{gathered} w_\nu^{\sigma\sigma'\alpha}(\mathbf{p}, \mathbf{k}, \mathbf{k}_1) = 2(2\pi)^6 \omega_1^2(k_1)\,\delta(\omega_2 - k_{2z}v_z - \nu\Omega_\alpha) \\ \times \left|\frac{\partial}{\partial\omega}\omega^2\varepsilon^\sigma\right|^{-1}_{\omega=\omega(\mathbf{k})} \left|\frac{\partial}{\partial\omega}\omega^2\varepsilon^{\sigma'}\right|^{-1}_{\omega=\omega_1(\mathbf{k}_1)} |a_i^*(\mathbf{k})\,\Lambda_{ij}(k, k_1)\,a_j(\mathbf{k}_1)|^2 \end{gathered} \tag{1.8}$$

$$\begin{gathered} \omega_2 = \omega - \omega_1, \qquad \mathbf{k}_2 = \mathbf{k} - \mathbf{k}_1, \qquad k = \{\mathbf{k}, \omega\} \\ \Lambda_{ij}(k, k_1) = \Lambda_{ij}^{(1)}(k, k_1) + \Lambda_{ij}^{(2)}(k, k_1) \\ \varepsilon^\sigma(k) = a_i(\mathbf{k})\,\varepsilon_{ij}(k)\,a_j(\mathbf{k}) + (\mathbf{k}\mathbf{a}(\mathbf{k}))(\mathbf{k}\mathbf{a}^*(\mathbf{k}))\,c^2\omega^{-2} \end{gathered} \tag{1.9}$$

$$\Lambda_{ij}(k, k_1) = [S_{ijs}(k, k_1, k_2) + S_{isj}(k, k_2, k_1)] E_s(k_2) \qquad (1.10)$$
$$E_s(k_2) = \Pi_{sl}(k_2) j_l(k_2)$$

Here $a(k^\sigma)$ is the unit polarization vector of the wave σ, e_α and Ω_α are the charge and cyclotron frequency of the particle of sort α, the tensor $\Lambda_{ij}^{(1)}(k, k_1)$ is connected with the particle vibrations in the wave field (Compton scattering), and $\Lambda_{ij}^{(2)}(k, k_1)$ is connected with the scattering of the incident wave by the screening charge cloud (nonlinear scattering proper). The components of the tensor $S_{ijs}(k, k_1, k_2)$ are found from the expressions for the nonlinear current in the plasma in a magnetic field [9], where the ion contribution is on the order of m_e/m_i of the electron concentration and can be neglected, and $\Pi_{sl}(k_2)$ is the inverse Maxwellian operator for the k_2 wave. The current $j(k_2)$ is determined by the unperturbed motion of the scattering particle in the magnetic field

$$j_l(k) = \sum_{\nu\alpha} \frac{e_\alpha}{(2\pi)^3} \delta(\omega - k_z v_z - \nu\Omega) \exp(-i\nu\varphi) \Gamma_l \qquad (1.11)$$

$$\Gamma_x = \frac{v_\perp}{2} [J_{\nu+1}(k_\perp r_\alpha) e^{-i\varphi} + J_{\nu-1}(k_\perp r_\alpha) e^{i\varphi}], \quad \Gamma_y = \frac{iv_\perp}{2} [J_{\nu-1}(k_\perp r_\alpha) e^{i\varphi} - J_{\nu+1}(k_\perp r_\alpha) e^{-i\varphi}]$$

$$\Gamma_z = v_z J_\nu(k_\perp r_\alpha), \qquad r_\alpha = v_\perp / \Omega_\alpha, \qquad \sin\varphi = k_y / k_\perp$$

The expressions for $\Lambda_{ij}^{(1)}$ are presented in [9].

In the problem in question some expressions can be simplified. First, the coefficients S_{ijs} in the quadratic term of the expansion of the nonlinear current in the interacting wave amplitudes, through which the scattering probability is expressed, can be expanded into a series in $\mu \ll 1$ and $\beta_n^{-1} \ll 1$, since it is precisely in this approximation (cold plasma approximation) that Eqs. (1.2) for the high frequency longitudinal oscillations in a magnetic field were obtained. This expansion into series in expressions for S_{ijs} corresponds to the expansion into a series in kv/ω in the kinetic equation for the electron distribution function of the second approximation, from which the expressions for S_{ijs} were obtained. The approximate expressions for the nonlinear currents have the form

$$S_{1jl}^{(e)} = -\frac{|e|\omega}{\omega^2 - \Omega^2} \frac{\varepsilon_{sl}^{(e)}(k_2) - \delta_{sl}}{4\pi m_e} \left\{k_{2s}\left(\delta_{j1} - i\frac{\Omega}{\omega}\delta_{j2}\right) + \frac{\omega_2}{\omega_1}\left[\delta_{js}\left(k_{1x} - i\frac{\Omega}{\omega}k_{1y}\right) - k_{1s}\left(\delta_{j1} - i\frac{\Omega}{\omega}\delta_{j2}\right)\right]\right\}$$

$$S_{2jl}^{(e)} = -\frac{|e|\omega}{\omega^2 - \Omega^2} \frac{\varepsilon_{sl}^{(e)}(k_2) - \delta_{sl}}{4\pi m_e} \left\{k_{2s}\left(\delta_{j2} + i\frac{\Omega}{\omega}\delta_{j1}\right) + \frac{\omega_2}{\omega_1}\left[\delta_{js}\left(k_{1y} + i\frac{\Omega}{\omega}k_{1x}\right) - k_{1s}\left(\delta_{j2} + i\frac{\Omega}{\omega}\delta_{j1}\right)\right]\right\}$$

$$S_{3jl}^{(e)} = -\frac{|e|}{\omega} \frac{\varepsilon_{sl}^{(e)}(k_2) - \delta_{sl}}{4\pi m_e} \left\{k_{2s}\delta_{j3} + \frac{\omega_2}{\omega_1}(k_{1z}\delta_{js} + k_{1s}\delta_{j3})\right\} \qquad (1.12)$$

Second, we can simplify the expressions for the tensor $\varepsilon_{ij}^{(e)}(k_2)$, where $k_2 = \{k_2, \omega_2\}$ is the virtual wave. In fact, $\omega_2 = \omega - \omega_1$, $k_2 = k - k_1$ (ω, k are the frequency and wave vector of the high frequency longitudinal wave, and ω_1, k_1 are the frequency and wave vector of the electron cyclotron wave $\omega_1 \approx \nu_0\Omega$). Since the absence of absorption of the electron cyclotron waves by the thermal particles in the plasma is associated with the perpendicularity of their propagation with respect to the direction of the external magnetic field (which was taken along the z axis), for the virtual wave $k_{2z} = k_z$. Therefore, in the expressions for $\varepsilon_{ij}^{(e)}(k_2)$ we can make an expansion in the parameter

$$\frac{\omega_2 - n\Omega}{|k_{2z}| v_e} = \frac{\omega - \omega_1 - n\Omega}{|k_z| v_e} = \frac{\omega - (n + \nu_0)\Omega}{|k_z| v_e} \gg 1 \qquad (1.13)$$

by virtue of (1.3) (the presence of the small correction $\Delta = \omega_1 - \nu_0\Omega$, $\Delta \ll \Omega$ does not alter the essence of the situation). In this approximation of the tensor $\varepsilon_{ij}(k_2)$ ($i, j = 1, 2, 3$) has the form

$$\varepsilon_{11} = 1 - \sum_n B_n(\omega_2)\left[\frac{n^2 A_n(\mu_2)}{\mu_2} - 2\mu_2 A_n'(\mu_2)\sin^2\varphi_2\right] \qquad (1.14)$$

$$\varepsilon_{22} = 1 - \sum_n B_n(\omega_2)\left[\frac{n^2 A_n(\mu_2)}{\mu_2} - 2\mu_2 A_n'(\mu_2)\cos^2\varphi_2\right]$$

$$\varepsilon_{12} = \varepsilon_{21}^* = ig = \sum_n iB_n(\omega_2) A_n'(\mu_2)(n + 2i\mu_2\sin\varphi_2\cos\varphi_2)$$

$$\varepsilon_{33} = 1 - \sum_n B_n(\omega_2) A_n(\mu_2), \qquad \varepsilon_{13} = \varepsilon_{31} = \varepsilon_{23} = \varepsilon_{32} = 0$$

$$A_n(\mu_2) = \exp(-\mu_2) I_n(\mu_2), \quad A_n' = \frac{dA_n}{d\mu_2}, \quad \mu_2 = \frac{k_{2\perp}^2 v_e^2}{\Omega^2}, \quad B_n = \frac{\omega_L^2}{\omega_2(\omega_2 - n\Omega)}$$

Here $I_n(\mu_2)$ is the modified Bessel function.

We note that expansion in the parameter μ_2 is not always possible, since although $\mu \ll 1$ for longitudinal waves, in the general case $\mu_1 < 1$, $\mu_1 > 1$ for electron cyclotron waves.

The expression for the inverse Maxwellian operator describing the virtual wave k_2 in the $\omega_2 - n\Omega \gg |k_{2z}| v_e$ approximation has the form

$$\Pi_{ij} = -\frac{4\pi i}{\omega_2}\frac{T_{ij}}{D} \qquad (i, j = 1, 2, 3) \tag{1.15}$$

$$\begin{aligned}
D &= N_2^4 [\varepsilon_{33}x^2 + (\varepsilon_{11}t^2 + \varepsilon_{22}(1-t^2))(1-x^2)] - N_2^2\{(1-x^2)[\varepsilon_{11}\varepsilon_{22} \\
&\quad - g^2 - \varepsilon_{11}\varepsilon_{33}(1-t^2) - \varepsilon_{22}\varepsilon_{33}t^2] + (\varepsilon_{11}+\varepsilon_{22})\varepsilon_{33}\} + (\varepsilon_{11}\varepsilon_{22} - g^2)\varepsilon_{33} \\
T_{11} &= N_2^4(1-x^2)t^2 - N_2^2\{\varepsilon_{22}(1-x^2) + \varepsilon_{33}[1-(1-x^2)(1-t^2)]\} + \varepsilon_{22}\varepsilon_{33} \\
T_{12} &= T_{21}^* = N_2^4(1-x^2)t\sqrt{1-t^2} - N_2^2\{\varepsilon_{33}(1-x^2)t\sqrt{1-t^2} - ig \times (1-x^2)\} - ig\varepsilon_{33} \\
T_{13} &= T_{31}^* = N_2^2 x\sqrt{1-x^2}\,[N_2^2 t + ig\sqrt{1-t^2} - \varepsilon_{22}t] \\
T_{22} &= N_2^4(1-x^2)(1-t^2) - N_2^2\{\varepsilon_{11}(1-x^2) + \varepsilon_{33}[1-(1-x^2)t^2]\} + \varepsilon_{11}\varepsilon_{33} \\
T_{23} &= T_{32}^* = N_2^2 x\sqrt{1-x^2}\,[N_2^2\sqrt{1-t^2} - \varepsilon_{11}\sqrt{1-t^2} - igt] \\
T_{33} &= N_2^4x^2 - N_2^2\{\varepsilon_{11}+\varepsilon_{22} - (1-x^2)[\varepsilon_{11}(1-t^2) + \varepsilon_{22}t^2]\} + \varepsilon_{11}\varepsilon_{22} - g^2
\end{aligned} \tag{1.16}$$

Here

$$\varepsilon_{mn} = \varepsilon_{mn}(k_2), \quad N_2^2 = k_2^2c^2/\omega_2^2, \; x^2 = k_{2z}^2/k_2^2, \quad t^2 = k_{2x}^2/k_{2\perp}^2 = \cos^2\varphi_2.$$

We shall now examine excitation of ordinary and quasilongitudinal cyclotron waves in the case of nonlinear scattering of the waves (1.4) (1.5) by the beam and plasma electrons.

2. EXCITATION OF ORDINARY ELECTRON CYCLOTRON WAVES

The dispersion equation for ordinary cyclotron waves has the form (the direction of the wave vector $\mathbf{k}_1$ is taken as the x axis)

$$n_1^2 = \varepsilon_{33}(k_1) = 1 - \frac{\omega_{L\alpha}^2}{\omega_1^2}\sum_{l=-\infty}^{\infty} A_l(\mu_{1\alpha})\frac{\omega_1}{\omega_1 - l\Omega_\alpha}, \qquad n_1^2 = \frac{k_1^2c^2}{\omega_1^2} \tag{2.1}$$

For the frequency ω_1 close to $\nu_0\Omega$, we have (ion motion may be neglected)

$$\frac{\omega_1 - \nu_0\Omega}{\nu_0\Omega} \approx -\frac{\varkappa}{\mu_1}A_{\nu_0}(\mu_1), \qquad \varkappa = q^2\frac{v_e^2}{c^2} \approx \frac{P_e}{P_H} \tag{2.2}$$

where $\varkappa$ is the ratio of the gaskinetic and magnetic pressures.

$$\left|\frac{\omega_1 - l\Omega}{\omega_1}\right| \gg \frac{v_e^2}{c^2} \qquad (l \text{ is an integer.}) \tag{2.3}$$

leads to the fact that propagation of ordinary cyclotron waves is possible only in a dense plasma $q \gg 1$ (for more detail on this see the dissertation of K. N. Stepanov, Khar'kov State University, 1965). We shall present the results of calculations of the nonlinear excitation increments of these waves for scattering of high frequency longitudinal waves with frequencies $\omega = \omega_\pm$ (1.4) by the plasma and beam electrons.

We shall examine two cases.

1. Long Waves, $\mu_1 \ll 1$. From (2.3) follows $\omega_L \gg \nu_0\Omega$. In this case $\mu_2 \ll 1$, since $\mu \ll 1$, and the nonzero components of the tensor $\varepsilon_{ij}^{(e)}(k_2)$ have the form

$$\varepsilon_{11} = \varepsilon_{22} = 1 - \frac{\omega_L^2}{\omega_2^2 - \Omega^2}, \quad \varepsilon_{33} = 1 - \frac{\omega_L^2}{\omega_2^2}, \quad g = \frac{\omega_L^2\Omega}{\omega_2(\omega_2^2 - \Omega^2)} \tag{2.4}$$

Evaluation of the inverse Maxwellian operator shows that in this case scattering takes place basically through the virtual longitudinal wave. The condition for this will be the inequality $N_2^2 \gg N_{2+}^2$, N_{2-}^2, where N_{2+}^2, N_{2-}^2 are roots of the equation $D = 0$. In the evaluation we used the conditions $n^2 = (kc/\omega)^2 \gg 1$ (condition of quasilongitudinality of the k waves), and $n_1^2 = (k_1c/\omega_1)^2 \gg 1$ follows from (2.1). The inverse Maxwellian operator takes the form

$$\Pi_{ij}(k_2) = -\frac{4\pi i}{\omega_2}\frac{k_{2i}k_{2j}}{k_2^2\varepsilon^l(k_2)} \qquad \left(\varepsilon^l(k_2) = \frac{k_{2i}k_{2j}}{k_2^2}\varepsilon_{ij}(k_2)\right) \tag{2.5}$$

For nonlinear scattering of the wave with frequency $\omega=\omega_+$ we find, using (1.10)-(1.12), (2.4), (2.5) with account for $\omega_L \gg \nu_0\Omega$

$$\Lambda^{(2)}(k, k_1) = a_i^*(\mathbf{k})\Lambda_{ij}^{(2)}(k, k_1)a_j(\mathbf{k}_1) \tag{2.6}$$

$$= -\frac{ie^2\omega_L}{2(2\pi)^3 m_e v_0^2\Omega^2}\left(\frac{k_1}{k_2}\right)^2 \cos\theta \sum_\nu \delta(\omega_2 - k_{2z}v_z - \nu\Omega) J_\nu(k_{2\perp}r)\exp i\nu\varphi_2$$

Compton scattering is defined by the expression

$$\Lambda^{(1)}(k, k_1) = a_i^*(\mathbf{k})\Lambda_{ij}^{(1)}(k, k_1)a_j(\mathbf{k}_1) = \frac{1}{(2\pi)^3}\frac{ie^2}{m_e k}\frac{\omega}{\omega_1} \tag{2.7}$$

$$\times \sum_\nu \delta(\omega_2 - k_{2z}v_z + \nu\Omega)\left[\frac{k_1 v_z}{(\omega_1-\nu\Omega)^2-\Omega^2}\left(k_x + ik_y\frac{\Omega}{\omega_1-\nu\Omega}\right) + \frac{k_z}{\omega_1-\nu\Omega}\right]J_\nu(k_1 r)$$

For

$$\frac{v_0\Omega}{\omega_1 - v_0\Omega} \gg \frac{k_1^2}{|\mathbf{k}-\mathbf{k}_1|^2}, \qquad \frac{k_z^2 u_e^2}{v_0^2\Omega^2} \gg 1,$$

Compton scattering is dominant. The scattering probability has the form (the resonant term in $\Lambda^{(1)}$ is retained)

$$w^{\sigma\sigma' e} = \frac{e^4}{2m_e^2k^2}\frac{J^2_{\nu_0}(k_1 r)}{A_{\nu_0}(\mu_1)}\delta(\omega - k_z v_z)\frac{\omega_L}{\omega_1\Omega^2}k_\perp^2\sin^2\theta \qquad \left(\frac{k}{k_1} \leqslant q\right) \tag{2.8}$$

The nonlinear excitation increment of the ordinary electron cyclotron waves is given by the formula

$$\gamma_{\mathbf{k}_1} \approx \frac{1}{2\sqrt{2\pi}}\frac{e^4}{m_e^3\omega_1}\frac{k_1^2}{\Omega^2}\int N_{\mathbf{k}}d\mathbf{k}\frac{\omega}{k^2|k_z|}\sin^2\theta\left\{\frac{\omega_2 n_0}{v_e^3}\exp\frac{-\beta_0^2}{2}\right. \tag{2.9}$$

$$\left. + \frac{\omega_2 - k_z v_0}{u_e^3}n_1\frac{A_{\nu_0}(\mu_1')}{A_{\nu_0}(\mu_1)}\exp\frac{-(\omega - k_z v_0)^2}{2k_z^2u_e^2}\right\} \qquad \left(\mu_1' = \frac{k_1^2u_e^2}{\Omega^2}\right)$$

The first term is associated with scattering by the plasma electrons; it is exponentially small. The second term is associated with scattering by the beam electrons. The buildup condition will be the presence of longitudinal k waves with negative projection k_z; otherwise, the beam introduces additional damping (prior to initiation of longitudinal waves buildup in the direction of smaller k), since

$$k_z v_0 > \omega_2 \tag{2.10}$$

This follows from the condition $k_z v_0 > \omega$ [6] for excitation by the beam of quasilongitudinal vibrations. However, buildup by waves with negative k_z is exponentially small, since the beam systematic velocity $v_0 > u_e$ (without account for increase of u_e by quasilinear relaxation). Buildup cyclotron waves is also possible by longitudinal waves with $k_z v_0 < \omega_2$ after spectral buildup of the initially excited waves with $k_z v_0 > \omega$ in the direction of smaller k. This occurs only for scattering with frequency reduction when $\omega > \omega_1$. The estimate of the maximal generation increment has the form

$$\gamma_{\mathbf{k}_1\max} \approx \frac{\sqrt{2\pi}}{8}\frac{W^l}{n_0T_e'}\frac{n_1}{n_0}\frac{A_{\nu_0}(\mu'_1)}{A_{\nu_0}(\mu_1)}\left(\frac{k_1}{k}\right)^2 q^2\frac{\omega_L^2}{v_0\Omega}$$

$$\left(T_e' = m_e u_e^2, \quad W^l = \int\omega N_{\mathbf{k}}\frac{d\mathbf{k}}{(2\pi)^3}\right) \tag{2.11}$$

for $k_z^2u_e^2 \gg v_0^2\Omega^2$, $\omega_2 - k_z v_0 \approx \omega - k_z v_0 \approx k_z u_e$

Here W^l is the total energy of the k waves. However, if for all the quasilongitudinal waves excited by the beam $(k_z u_e/\omega_1)^2 \ll 1$, the increment $\gamma_{\mathbf{k}_1}$ with account for only the resonance term in (2.7) is exponentially small

$$\gamma_{\mathbf{k}_1} \sim \frac{k_z u_e}{\omega_1}\exp\frac{-\omega_1^2}{2k_z^2u_e^2}\gamma_{\mathbf{k}_1\max}$$

With account for the remaining terms in (2.6), (2.7) $\gamma_{\mathbf{k}_1} \sim (\omega_1 - v_0\Omega)^2\omega_1^{-2}v_0^{-4}$ of (2.11).

Now let us examine excitation of the ordinary cyclotron wave for scattering of a longitudinal wave with frequency $\omega=\omega_-$; $\Lambda^{(2)}$ is given by the expression

$$\Lambda^{(2)} \approx -\frac{1}{(2\pi)^3}\frac{ie^2k_z}{m_e k\omega}\left\{1+\frac{k_1^2}{k_2^2}\frac{\omega_2^2}{\omega_1^2}+\frac{k_1\omega^2\omega_2}{k_2^2\omega_1(\omega^2-\Omega^2)}\left(k_x+ik_y\frac{\omega}{\Omega}\right)\right.$$
$$\left.\times\sum_\nu\delta(\omega_2-k_{2z}v_z-\nu\Omega)J_\nu(k_{2\perp}r)\exp i\nu\varphi_2\right\} \qquad (2.12)$$

The Compton scattering is given by (2.7), where $\omega=\omega_-$. In this case, since $\omega<\omega_1$ buildup is possible only in the presence of waves with negative k_z. It is exponentially small by virtue of $v_0 > u_e$ (with account for only the resonance term in (2.7)). The estimate of the maximal excitation increment of the ordinary cyclotron wave for dominant Compton scattering by the beam electrons has the form [the largest term in (2.7) satisfying the condition $\omega_2+k_zv_0+\nu_1\Omega\approx 0$ is retained]

$$\gamma_{\mathbf{k},\max}\approx\frac{\sqrt{2\pi}}{4}\frac{k_zv_0-\nu_0\Omega}{|k_z|u_e}\frac{W^l}{n_0T_e'}\frac{A_{\nu_1}(\mu_1')}{A_{\nu_0}(\mu_1)}\frac{n_1}{n_0}\frac{\omega^2}{\nu_0\Omega}\left(\frac{k_1}{k}\right)^2\left(\frac{m_i}{m_e}\right)^2\frac{(\omega_1-\nu_0\Omega)^2}{k^2v_0^2} \qquad (2.13)$$

Here we assumed $\Omega \ll k_zv_0 \ll v_0k_1 \operatorname{tg}\,\theta$.

This estimate was obtained at the limit of satisfaction of (1.6), i.e., for $\theta\sim\theta_{\max}$; W^l is the energy of the k waves in a narrow range of angles around $\theta_{\max}$.

2. Short Waves, $\mu_1 \gg 1$. We shall use (1.3). Then for the wave with frequency $\omega=\omega_-$ $\mu\ll 1$. For the wave with frequency $\omega=\omega_+$ we shall also assume this inequality is satisfied. In this case, in view of $\mu_1\gg 1$ we have $k_1 \gg k$, i.e., for the virtual wave $\mathbf{k}_2\approx-\mathbf{k}_1$, and in (1.14)-(1.16) we must set $\theta_2=\pi/2$, $\varphi_2=\pi$. Using (1.10)-(1.12), (1.14)-(1.16), we obtain

$$\Lambda^{(2)}(k,k_1)=\left(\frac{k_x}{k}+i\frac{k_y}{k}\frac{\Omega}{\omega}\right)\frac{|e|\omega}{\omega^2-\Omega^2}\frac{k_1}{4\pi m_e}\left[\frac{\omega_1}{\omega_2}(\varepsilon_{33}^{(e)}(k_1)-1)\right.$$
$$\left.-\frac{\omega_2}{\omega_1}(\varepsilon_{33}^{(e)}(k_2)-1)\right]E_{zk_2}+\frac{k_z}{k}\frac{|e|}{m_e\omega_1}\frac{k_1}{4\pi}\left[(\varepsilon_{11}^{(e)}(k_2)-1)E_{xk_2}+\varepsilon_{12}^{(e)}(k_2)E_{yk_2}\right] \qquad (2.14)$$

$$E_{xk_2}=-\frac{4\pi i}{\omega_2}\frac{1}{D}\left[(N_2^2-\varepsilon_{22})(N_2^2-N_{2+}^2)j_{xk_2}+ig(N_2^2-N_{2-}^2)j_{yk_2}\right]$$
$$E_{yk_2}=-\frac{4\pi i}{\omega_2}\frac{1}{D}\left[-ig(N_2^2-N_{2+}^2)j_{xk_2}-\varepsilon_{11}(N_2^2-N_{2+}^2)j_{yk_2}\right] \qquad (2.15)$$
$$E_{zk_2}=\frac{4\pi i}{\omega_2}\frac{1}{D}\varepsilon_{11}(N_2^2-N_{2-}^2)j_{zk_2}$$

$$D=\varepsilon_{11}(N_2^2-N_{2+}^2)(N_2^2-N_{2-}^2),\quad N_{2+}^2=\varepsilon_{33},\quad N_{2-}^2=\varepsilon_{22}-g^2/\varepsilon_{11} \qquad (2.16)$$

Comparison of (2.14) and (2.15), (2.16) makes it possible to conclude that the first term in (2.14) is determined by scattering through the virtual ordinary wave (a denominator of the form $N_2^2-N_{2+}^2$ remains), while the second term is determined by scattering through a combination of the virtual extraordinary wave and the virtual longitudinal wave (if such can be identified).

In the general case, we retain from the entire sum the term which includes $\varepsilon_{33}^{(e)}(k_1)-1$, since this includes the term with the resonance denominator $\omega_1-\nu_0\Omega$. Taking into account the dispersion equation for the ordinary cyclotron wave, we can write

$$\Lambda^{(2)}(k,k_1)=-\frac{ie^2\omega}{\omega^2-\Omega^2}\frac{k_1v_z}{m_e\omega_1}\frac{k_1^2c^2}{\omega_2^2}\frac{1}{N_2^2-N_{2+}^2}\frac{1}{(2\pi)^3}\times\left(\frac{k_x}{k}+i\frac{k_y}{k}\frac{\Omega}{\omega}\right)\sum_\nu J_\nu(k_{2\perp}r)\,\delta(\omega_2-k_{2z}v_z+\nu\Omega) \qquad (2.17)$$

The Compton scattering is defined by (2.7).

Identifying in (2.17) the resonance term and comparing with (2.17), we conclude that the Compton scattering mechanism is definitive provided

$$\frac{\omega_1-\nu\Omega}{\Omega}\ll\frac{N_2^2-N_{2+}^2}{N_2^2} \qquad (2.18)$$

Since $(\omega_1-\nu_0\Omega)/\Omega\ll 1$ (satisfaction of this condition is what permits considering propagation of the cyclotron harmonic) and the k_2 wave is virtual and not the actually propagating ordinary wave (only for which is the condition $N_2^2=N_{2+}^2$ satisfied), then (2.18) can be considered met.

In this case the estimates for the ordinary cyclotron wave generation increments coincide with the estimates for $\mu_1 \ll 1$ with satisfaction of the same buildup conditions (remarks concerning (2.10)) and are given by (2.9), (2.11) for transformation of the quasilongitudinal wave with frequency $\omega=\omega_+$ and by (2.13) for transformation of the wave with frequency $\omega=\omega_-$ with account for the fact that

$$A_{\nu_1}(\mu_1')\,/\,A_{\nu_0}(\mu) \approx v_e\,/\,u_e \quad \text{for} \quad \mu_1 \gg 1, \quad \mu_1' \gg 1$$

3. EXCITATION OF QUASILONGITUDINAL CYCLOTRON WAVES

From the equation for the extraordinary cyclotron wave

$$\varepsilon_{11}(k_1)n_1^2 - \varepsilon_{11}(k_1)\varepsilon_{22}(k_1) - \varepsilon_{12}^2(k_1) = 0 \tag{3.1}$$

for $n_1^2 \gg \varepsilon_{22}(k_1) + \varepsilon_{12}^2(k_1)/\varepsilon_{11}(k_1)$ we can obtain the equation for the quasilongitudinal cyclotron wave

$$\varepsilon_{11}(k_1) = 1 - \frac{\omega_L^2}{\omega_1^2}\sum_{l=-\infty}^{\infty}\frac{l^2 A_l(\mu_{1\alpha})}{\mu_{1\alpha}}\frac{\omega_1}{\omega_1 - l\Omega} = 0 \tag{3.2}$$

For a frequency ω_1 which is close to $\nu_0\Omega$, neglecting ion motion, we have (the resonance term and terms with $l = \pm 1$ are retained)

$$\frac{\omega_1 - \nu_0\Omega}{\nu_0\Omega} \approx \frac{A_{\nu_0}(\mu_1)}{\mu_1}\left[\frac{\Omega^2}{\omega_L^2} - \frac{2}{\nu_0^2 - 1}\frac{A_1(\mu_1)}{\mu_1}\right]^{-1} \tag{3.3}$$

Following are the results of calculations of the nonlinear excitation increments of these waves for dense and low-density plasmas.

1. Dense Plasma, $q \gg 1$. It follows from (3.3) that propagation of quasilongitudinal waves with frequencies which are multiples of the electron cyclotron frequency is possible in a dense plasma only for $\mu_1 \ll 1$ [for $\mu_1 \gtrsim 1$ the wave frequencies are far from $\nu_0\Omega$, which contradicts the assumption adopted in deriving (3.3)]. For $\mu_1 \ll 1$, as in the case of the ordinary cyclotron wave, we can use (2.4), (2.5) and assume that nonlinear scattering by the virtual longitudinal wave takes place.

It can be shown that in transformation of the wave with frequency $\omega=\omega_+$ into the quasilongitudinal cyclotron wave Compton scattering is dominant and is defined by the expression

$$\Lambda^{(1)}(k, k_1) = \frac{1}{(2\pi)^3}\frac{ie^2}{km_e}\sum_\nu \delta(\omega_2 - k_{2z}v_z + \nu\Omega)J_\nu(k_1 r)\frac{\omega}{(\omega_1 - \nu\Omega)^2 - \Omega^2} \times \left(k_x + i\frac{\Omega}{\omega_1 - \nu\Omega}k_y\right) \tag{3.4}$$

Retaining the resonance term in the sum (3.4), we write the expression for the nonlinear increment

$$\gamma_{\mathbf{k}_1} \approx \frac{1}{2\sqrt{2\pi}}\frac{e^4}{m_e^3\Omega^2}\frac{\mu_1}{\nu_0 q}\int N_{\mathbf{k}}\,d\mathbf{k}\frac{k_\perp^2}{k^2|k_z|}\left\{\frac{n_0\omega_2}{v_e^3}\exp\left(-\frac{\beta_0^2}{2}\right)\right.$$
$$\left. + \frac{\omega_2 - k_z v_0}{u_e^3}n_1\frac{A_{\nu_0}(\mu_1')}{A_{\nu_0}(\mu_1)}\exp\frac{-(\omega - k_z v_0)^2}{2k_z^2 u_e^2}\right\} \tag{3.5}$$

If buildup takes place (remark to (2.10), (2.11)), the maximal increment has the estimate

$$\gamma_{\mathbf{k}_1\max} \approx \frac{\sqrt{2\pi}}{4}\frac{n_1}{n_0}\frac{W^l}{n_0 T_e'}\frac{\omega_L^2}{\nu_0\Omega}\mu_1\frac{A_{\nu_0}(\mu_1')}{A_{\nu_0}(\mu_1)} \tag{3.6}$$

$$\omega - k_z v_0 \approx k_z u_e, \quad k_1^2 u_e^2 \gg \nu_0^2\Omega^2$$

Compton scattering also dominates in transformation of the wave with frequency $\omega=\omega_-$ into the quasilongitudinal cyclotron wave. The estimate for the maximal increment has the form at the limit of satisfaction of (1.6)

$$\gamma_{\mathbf{k}_1\max} \approx \frac{\sqrt{2\pi}}{4}\frac{n_1}{n_0}\frac{W^l}{n_0 T_e'}\mu_1\left(\frac{\Omega}{kv_0}\right)^4\frac{(\omega_1 - \nu_0\Omega)^2}{\nu_0\Omega}\frac{k_z v_0 - \nu_0\Omega}{|k_z|u_e}\frac{A_{\nu_1}(\mu_1')}{A_{\nu_0}(\mu_1)}\left(\frac{m_i}{m_e}\right)^2 \tag{3.7}$$

$$(\omega_2 + k_z v_0 + \nu_1\Omega \approx 0)$$

2. Nondense Plasma, $q \ll 1$. Propagation of quasilongitudinal waves with frequencies close to the harmonics of the electron cyclotron frequency is possible for both $\mu_1 \ll 1$ and $\mu_1 \gg 1$.

In the case $\mu_1 \ll 1$ for $\varepsilon_{ij}(k_2)$ we can use Eqs. (2.4) and assume that nonlinear scattering takes place through the virtual longitudinal wave.

Nonlinear scattering of the wave with frequency (1.5) $\omega = \omega_+$ is defined by the quantity

$$\Lambda^{(2)}(k, k_1) = \frac{ie^2}{(2\pi)^3 m_e k} \frac{k_1 \Omega}{\omega_L^2 \sin^2\theta} \frac{k_\perp^2 - (k_x + ik_y) k_1}{|\mathbf{k} - \mathbf{k}_1|^2} \sum_\nu J_\nu(k_{2\perp} r)\, \delta(\omega_2 - k_{2z} v_z - \nu\Omega) \exp i\nu\varphi_2 \tag{3.8}$$

Here it is assumed that

$$\omega^2 k_{2\perp} k_\perp \gg k_z^2 (\omega^2 - \Omega^2), \quad k_1^2 \gg k_2^2 q (\nu_0 - 1)^2$$

Expression (3.4), in which $\omega \approx \Omega$, remains valid for Compton scattering.

For both this wave and the wave with frequency $\omega = \omega_-$, by virtue of the inequality $\omega < \omega_1 \approx \nu_0 \Omega$ in the case of their scattering with transformation into electron cyclotron waves, buildup of the latter is possible only in the presence of waves with negative k_z. Account for only the resonance term in (3.4) leads to an exponentially small buildup increment. Therefore, we must evaluate the contribution of the remaining terms in (3.4) and (3.8) to the nonlinear increment. The corresponding estimate shows that the main contribution is made by the largest term in (3.8), satisfying the condition $\omega_2 + k_z v_0 - \nu_1 \Omega \approx 0$ (for $k_z u_e/\Omega \ll 1$). The expression for the nonlinear increment for scattering of longitudinal waves with $\omega = \omega_+$ has the form

$$\gamma_{k_1} \approx \frac{1}{\sqrt{2\pi}} \frac{e^4}{m_e^3} \mu_1 k_1^2 \frac{(\omega_1 - \nu_0\Omega)^2 \Omega}{\omega_1 \omega_L^4} \int N_{\mathbf{k}}\, d\mathbf{k} \frac{k_{2\perp}^4}{k_2^4 |k_z|}$$
$$\times \left\{ \frac{n_0 \omega_2}{v_e^3} \frac{A_{\nu_1}(\mu_1)}{A_{\nu_0}(\mu_1)} \exp \frac{-v_0^2}{2v_e^2} + \frac{n_1}{u_e^3} (\omega_2 + k_z v_0) \frac{A_{\nu_1}(\mu_1')}{A_{\nu_0}(\mu_1)} \right\} \tag{3.9}$$

We have assumed that

$$\frac{k_1^2 k_{2\perp}^2}{k_2^4} \frac{(k_z v_0)^4}{\omega_L^4 \sin^4\theta} \gg 1$$

For the estimate of the maximal increment we obtain

$$\gamma_{k_1 \max} \approx \frac{\sqrt{2\pi}}{4} \frac{n_1}{n_0} \frac{W^l}{n_0 T_e'} \frac{A_{\nu_1}(\mu_1')}{A_{\nu_0}(\mu_1)} \frac{k_1^2}{|\mathbf{k} - \mathbf{k}_1|^2} \mu_1 \frac{(\omega_1 - \nu_0\Omega)^2}{\omega_1} \frac{k_z v_0 - \nu_0 \Omega}{|k_z| u_e} \tag{3.10}$$

For $k_z u_e/\Omega \gtrsim 1$ several terms in (3.8) make the same contribution to γ_{k_1}.

Similarly, for scattering of the wave with frequency $\omega = \omega_-$, retaining the largest term in $\Lambda^{(2)}$, we obtain the estimate for the maximal increment (for $k_z^2 > k_\perp k_1 q$)

$$\gamma_{k_1 \max} \approx \frac{\sqrt{2\pi}}{4} \frac{n_1}{n_0} \frac{W^l}{n_0 T_e'} \frac{k_1^2 k^2}{|\mathbf{k} - \mathbf{k}_1|^4} \mu_1 \frac{A_{\nu_1}(\mu_1')}{A_{\nu_0}(\mu_1)} \frac{(\omega_1 - \nu_0\Omega)^2}{q^2 \omega_1} \frac{k_z v_0 - \nu_0\Omega}{|k_z| u_e} \tag{3.11}$$

By analogy with Section 2.2, for $\mu_1 \gg 1$, $k_1 \gg k$, i.e., $\mathbf{k}_2 \approx -\mathbf{k}_1$, for both the wave with frequency $\omega = \omega_+$ and the wave with frequency $\omega = \omega_-$. In the general case the nonlinear scattering proper is defined by the quantity

$$\Lambda^{(2)}(k, k_1) = \left(\frac{k_x}{k} + i\frac{k_y}{k}\frac{\Omega}{\omega}\right) \frac{|e|\omega}{\omega^2 - \Omega^2} \frac{k_1}{4\pi m_e} [(\varepsilon_{11}^{(e)}(k_2) - 1)$$
$$- (\varepsilon_{11}^{(e)}(k_1) - 1)] E_{xk_2} + \left[\left(\frac{k_x}{k} + i\frac{k_y}{k}\frac{\Omega}{\omega}\right)\left(\varepsilon_{12}^{(e)}(k_2) + \frac{\omega_1}{\omega_2}\varepsilon_{21}^{(e)}(k_1)\right)\right.$$
$$\left. + \left(i\frac{\Omega}{\omega}\frac{k_x}{k} - \frac{k_y}{k}\right)\frac{\omega}{\omega_2}(\varepsilon_{11}^{(e)}(k_1) - 1)\frac{|e|\omega}{\omega^2 - \Omega^2}\frac{k_1}{4\pi m_e}\right] E_{yk_2} - \frac{k_z}{k}\frac{|e| k_1}{\omega_2}\frac{\varepsilon_{11}^{(e)}(k_1) - 1}{4\pi m_e} E_{zk_2} \tag{3.12}$$

Here E_{sk_2} are given by (2.15), (2.16).

Expression (3.4) holds for Compton scattering. Buildup is possible only in the presence of waves with negative k_z, where account for the resonance term in (3.4) leads to an exponentially small increment. The primary contribution will be made by the largest terms in (3.4) and (3.12), satisfying the condition $\omega_2 + k_z v_0 + \nu_1 \Omega \approx 0$ for $k_z u_e/\Omega \ll 1$.

If the virtual wave is longitudinal, i.e., $N_2^2 \gg N_{2+}^2, N_{2-}^2$, then $\Lambda^{(2)}$ has the form (neglecting $\varepsilon_{11}^{(e)}(k_2) - 1$ by virtue of the resonance denominator in $\varepsilon_{11}^{(e)}(k_1) - 1$ and assuming $\varepsilon_{11}^{(e)}(k_1) = 0$)

$$\Lambda^{(2)}(k, k_1) \approx -\frac{ie^2\omega}{\omega^2-\Omega^2}\frac{1}{(2\pi)^3 m_e \varepsilon_{11}(k_2)}\left(\frac{k_x}{k}+i\frac{k_y}{k}\frac{\Omega}{\omega}\right)\sum_\nu \delta(\omega_2-k_{2z}v_z+\nu\Omega)J_\nu(k_{2\perp}r) \quad (3.13)$$

Comparison of the largest terms in (3.4) and (3.13) leads to the conclusion that nonlinear scattering dominates if

$$(\omega^2-\Omega^2)\varepsilon_{11}(k_2) < k_z^2 v_0^2 \ (k_z v_0 > \Omega)$$

The maximal increment estimate for scattering of waves with frequencies $\omega=\omega_+$, $\omega=\omega_-$ is

$$\gamma_{k_1\max} \approx \frac{\sqrt{2\pi}}{4}\frac{n_1}{n_0}\frac{W^l}{n_0 T_e'}\frac{v_e}{u_e}\frac{\mu_1}{|\varepsilon_{11}(k_2)|^2}\frac{(\omega_1-\nu_0\Omega)^2}{\omega_1}\frac{k_z v_0-\nu_0\Omega}{|k_z|\,u_e} \quad (3.14)$$

However, for a univariate distribution $k=k_z$ for the wave with frequency $\omega=\omega_-$ the scattering is completely defined by the last term in (3.12). For the wave with frequency $\omega=\omega_+$ this angle span is excluded from consideration, since $\omega_+\to\Omega$ as $\theta\to 0$.

For comparison, we write the condition for domination of nonlinear scattering if it takes place through the virtual ordinary wave; $\Lambda^{(2)}$ has the form

$$\Lambda^{(2)}(k, k_1) \approx -\frac{ie^2 k_z k_1}{(2\pi)^3 k m_e \omega_2^2}\frac{v_z}{N_2^2-N_{2+}^2}\sum_\nu \delta(\omega_2-k_{2z}v_z+\nu\Omega)J_\nu(k_{2\perp}r) \quad (3.15)$$

and nonlinear scattering proper dominates if

$$\frac{N_2^2}{N_2^2-N_{2+}^2} \gg \frac{\omega}{(k_z v_0)^3}\frac{k_\perp}{k_1}|\mathbf{k}-\mathbf{k}_1|^2 c^2 \quad (k_z v_0 > \Omega) \quad (3.16)$$

4. SOME ESTIMATES

Limiting ourselves to the calculations presented above for the nonlinear excitation increments of the ordinary and quasilongitudinal cyclotron waves (the expressions for the extraordinary cyclotron waves are not presented because of their complexity, particularly since for $\varkappa \ll \mu_1 \ll 1$ the corresponding extraordinary wave branch becomes plasma waves), we note the following characteristic features of the nonlinear cyclotron wave generation mechanism.

Excitation of cyclotron waves is possible in both dense and nondense plasmas. In the dense plasma generation can start either after several cycles of nonlinear transfer of the quasilongitudinal waves excited by the beam across the spectrum toward smaller k, when the condition $\omega-\omega_1-k_z v_0>0$ begins to be satisfied, or after isotropization of the quasilongitudinal waves because of different nonlinear isotropization mechanisms. The ordinary waves are excited more intensely [compare (2.11) and (3.7) for $\mu_1 \ll 1$]. However, we note that the increments will be quite large only for scattering of waves with $\omega=\omega_+$ and satisfaction of the conditions

$$\omega_1 \leqslant k_z u_e < k_z v_0 < \omega_L - \omega_1,\ \omega_1 \approx \nu_0\Omega \leqslant \omega_L \quad (4.1)$$

Thus, in the dense plasma the first harmonics of the electron gyrofrequency are excited most intensely. The excitation increment of the higher harmonics ($\omega_1 \gg k_z u_e$) is several orders smaller, and excitation of the frequencies $\nu_0\Omega > \omega_L$ is possible only in the presence of negative k_z and is also several orders smaller than (2.11). Similarly, excitation of the electron cyclotron harmonics for scattering of waves with $\omega=\omega_-$ is possible only in the presence of negative k_z and in magnitude is several orders smaller than (2.11).

In the nondense plasma only the quasilongitudinal cyclotron waves are excited. Their nonlinear excitation is possible in the presence of negative k_z. It follows from the estimates (3.10), (3.11), (3.14), that this excitation is also several orders less intense than (2.11).

Let us estimate the characteristic generation time for ordinary cyclotron waves for $\mu_1 \gg 1$ on the basis of (2.11). We set $\omega_L\sim 10\Omega$, $v_e\sim 10^{-2}u_e\sim 10^{-3}v_0$, $W^l\sim 10^{-3}n_0 T_e$, $n_1\sim 10^{-3}n_0$, $k_1 v_e\sim 10\Omega$, $kv_0\sim \omega_L$.

In this case $\gamma_{k_1\max}\sim 10^{-6}\,\omega_L \sim 10^{-4}\ \text{sec}^{-1}$ for $n_0\sim 10^{12}\ \text{cm}^{-3}$.

However, we must recall that nonlinear generation can occur only after nonlinear transfer of the longitudinal waves excited by the beam across the spectrum in the direction toward smaller k.

Let us evaluate the time for transfer from the values $k_z v_0 \gtrsim \omega$ to the values $k_z v_0 \lesssim \omega_2$, i.e., by $\omega - \omega_2 = \omega_1 \approx \nu_0 \Omega$. In the case of transfer with ion scattering the estimate of the maximal transfer increment has the form

$$\gamma_{\mathbf{k}\,\max} \approx \frac{\sqrt{2\pi}}{8} \omega_L \frac{W^l}{n_0 T_i} \left(1 + \frac{T_e}{T_i}\right)^{-2} \tag{4.2}$$

for transfer through $\Delta\omega \approx |\Delta k_z^l| v_i \approx v_i \omega_L^* / v_0$ (for $\theta \sim 1$, where θ is the angle between k^l and k^l_1).

For $W^l \sim 10^{-2} n_0 T_i$, $T_e \sim 10 T_i$, $v_0 \sim 10^4 v_i$ the time for transfer through $\omega - \omega_2$

$$\tau \approx \frac{8}{\sqrt{2\pi}} \frac{n_0 T_i}{W^l} \omega_L^{-1} \left(1 + \frac{T_e}{T_i}\right)^2 \nu_0 \frac{v_0}{v_i q}$$

is of the same order as (2.11).

The other increments discussed above can be evaluated similarly.

Both ordinary and quasilongitudinal waves can be excited in the dense plasma for $\mu_1 \gg 1$. In the non-dense plasma only the quasilongitudinal waves can be excited.

As indicated above, the linear generation mechanism does not operate if the beam electron distribution function is Maxwellian; for transverse velocity, distribution in the form of the δ-function excitation of the cyclotron harmonics is missing for $\lambda < 1$ [3-5].

For comparison we write the expression for the nonlinear ordinary electron cyclotron wave generation increment for scattering of quasilongitudinal waves with

$$\omega \approx \omega_L + \frac{\Omega^2}{2\omega_L} \sin^2\theta$$

by beam electrons with the distribution function

$$f_0(\mathbf{v}) = n_1 (2\pi)^{-3/2} v_{0\perp}^{-1} u_e^{-1} \delta(v_\perp - v_{0\perp}) \exp \frac{-(v_z - v_0)^2}{2u_e^2}$$

[assuming, as in obtaining (2.9), that Compton scattering is dominant, and retaining the resonance term in (2.7)]. In this case the plasma electron contribution to $\gamma_{\mathbf{k}_1}$ is exponentially small.

$$\gamma_{\mathbf{k}_1} \approx \frac{\sqrt{2\pi}}{8} \frac{n_1}{n_0} \frac{1}{n_0 T_e'} \frac{\omega_L^5}{\omega_1 \Omega^2} \frac{k_1^2}{A_{\nu_0}(\mu_1)} \int N_{\mathbf{k}} \frac{d\mathbf{k}}{(2\pi)^3} \frac{\sin^2\theta}{|k_z|^3} \exp \frac{-(\omega - k_z v_0)^2}{2k_z^2 u_e^2}$$
$$\times \left\{ 2J_{\nu_0}(\lambda) J_{\nu_0}'(\lambda) \nu_0 \frac{k_1 u_e}{v_{0\perp}} - \frac{\omega - k_z v_0}{u_e} J_{\nu_0}^2(\lambda) \right\} \tag{4.3}$$

Hence we see that $\gamma_{\mathbf{k}_1}$ changes sign when the quantity λ passes through the roots of the expression in the braces, i.e., ranges of stability and instability arise, associated with the form of the transverse velocity distribution function, as in linear theory.

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